



K21P 4212

Reg. No. :

Name :

I Semester M.Sc. Degree (CBSS – Reg./Supple./Imp.)
Examination, October 2021
(2018 Admission Onwards)
MATHEMATICS
MAT1C04 : Basic Topology

Time : 3 Hours

Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks.

1. Let $X = \{a, b, c\}$. Give an example of a collection of subsets of X which is a topology on X . Further, give an example of a collection of subsets of X which is not a topology on X .
2. Is $\text{int}(A \cup B) = \text{int}(A) \cup \text{int}(B)$? Justify your answer.
3. Describe the weak topology on \mathbb{R} induced by the family of constant functions from \mathbb{R} to \mathbb{R} , where the co-domain has the usual topology ? Justify your answer.
4. Let (A, τ_A) be a subspace of (X, τ) . Is a set open in (A, τ_A) be necessarily open in (X, τ) ? Justify your answer.
5. Prove that the closed unit interval has the fixed point property.
6. Is connectedness a hereditary property ? Justify your answer.

PART – B

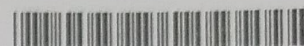
Answer **any four** questions from this Part without omitting **any** Unit. **Each** question carries **16** marks.

Unit – I

7. a) Define finite complement topology on a set X . Show that finite complement topology is a topology on X .
b) Let $X = \{a, b, c\}$ and $B = \{\{a, b\}, \{b, c\}, X\}$. Can B be a basis for a topology on X . Justify your answer.
c) Give an example of a basis B for a space X . Show that this B satisfies the conditions for a collection of sets to be a basis.

P.T.O.





8. a) Let X be a set and S be a collection of subsets such that $X = \bigcup \{S : S \in S\}$. Prove that there is a unique topology τ on X for which S is a sub basis.
- b) Is \mathbb{R} with finite complement topology a first countable space? Justify your answer.
- c) Prove that every second countable space is separable.
9. a) Show that, in a Hausdorff space a convergent sequence has a unique limit.
- b) Let (X, d) be a metric space, $\langle x_n \rangle$ a Cauchy sequence in X and let $A = \{x_n : n \in \mathbb{N}\}$. Prove that A is bounded.
- c) Let (X, τ) be a topological space, (Y, d) a metric space, $f : X \rightarrow Y$ a function and $f_n : X \rightarrow Y$ a continuous function for each $n \in \mathbb{N}$ such that $\langle f_n \rangle$ converges uniformly to f . Prove that f is continuous.

Unit – II

10. a) Define subspace topology on A , where A is a subset of a topological space X . Show that subspace topology is a topology on the subset A .
- b) Is separability a hereditary property? Justify your answer.
- c) Let (X, τ) , (Y_1, U_1) , (Y_2, U_2) be topological spaces. Prove that $f : X \rightarrow Y_1 \times Y_2$ is continuous if and only if $\pi_i \circ f$ is continuous for each $i = 1, 2$.
11. a) Let (X_1, τ_1) , (X_2, τ_2) be topological spaces and $(X_1 \times X_2, \tau)$ be the product space. Show that the product topology on $X_1 \times X_2$ is the smallest topology for which both the projections, from the product space to the factor spaces, are continuous.
- b) Let (X_1, d_1) and (X_2, d_2) be metric spaces and let $X = X_1 \times X_2$. Prove that the product topology on X is same as the topology on X generated by the product metric.
- c) Define weak topology. Define product topology for an arbitrary collection of topological spaces in terms of weak topology.